GENERALIZED KROM SPACES AND THE MENGER GAME

MATHEUS DUZI

Back when it was still unknown whether there was an example in ZFC of a Baire spaces which is not productively Baire (a question that was later answered by Cohen with an example using forcing), Krom showed that the existence of any such example implied the existence of an ultrametric example. In order to do this, Krom introduced what is now called *the Krom space of a given space* X, which is an ultrametric hyperspace constructed from X.

The Krom space of X is closely related to the Banach-Mazur game over X. It is inspired by this connection that we generalize the construction for any infinite game G. We are thus able to show that an infinite game G is *isomorphic* (in some sense) to a subgame of the Banach-Mazur game over its Krom space $\mathsf{K}(G)$. Furthermore, we show that the Menger game is equivalent to the Banach-Mazur game over a certain Krom space and, hence, by combining Hurewicz Theorem and Oxtoby's Theorem, we show that a space X is Menger if, and only if, said Krom space is Baire.

References

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(Matheus Duzi) ICMC-USP
Email address: matheus.duzi.costa@usp.br